

Note: - You have four choices for each objective type question as A, B, C and D. The choice which you think is correct; fill that circle in front of that question number in your answer book. Use marker or pen to fill the circles. Cutting or filling up two or more circles will result no mark.

| Q.1 | Questions | A | B | C | D |
|-----|---|---------------------------|-----------------------------------|----------------------------------|---------------------------------|
| 1. | The conic is called circle if: | $e = 1$ | $e < 1$ | $e = 0$ | $e > 1$ |
| 2. | The direction cosines of Z -axis are: | $(1, 0, 0)$ | $(0, 1, 0)$ | $(0, 0, 1)$ | $(0, 0, 0)$ |
| 3. | Angle between nonzero vectors \underline{a} and $\underline{a} \times \underline{b}$ is: | 0 | $\frac{\pi}{6}$ | $\frac{\pi}{3}$ | $\frac{\pi}{2}$ |
| 4. | Distance between $(-1, 2)$ and $(7, 5)$ is: | ± 7 | $\sqrt{73}$ | 73 | $\sqrt[2]{73}$ |
| 5. | Opening of parabola $x^2 = -16y$ is: | downward | upward | left | right |
| 6. | If \underline{u} and \underline{v} are parallel vectors having same direction then $\underline{u} \cdot \underline{v}$ is equal to: | $-uv$ | uv | $uv \sin \theta$ | $uv \tan \theta$ |
| 7. | If $f(x) = \sqrt{x+4}$ then $f(x-1)$ is equal to: | $\sqrt{x+4}$ | $\sqrt{x+3}$ | $\sqrt{x+2}$ | $\sqrt{x+1}$ |
| 8. | The distance of point $(1, -2)$ from Y -axis is: | 2 | 3 | 4 | 1 |
| 9. | Vertices of $\frac{y^2}{16} - \frac{x^2}{49} = 1$ are: | $(\pm 4, 0)$ | $(0, \pm 4)$ | $(0, \pm 7)$ | $(\pm 7, 0)$ |
| 10. | Inclination of line perpendicular to Y -axis is: | $\frac{\pi}{3}$ | $\frac{\pi}{6}$ | $\frac{\pi}{2}$ | zero |
| 11. | $\frac{d}{dx}(\sin^2 x + \cos^2 x)$ is equal to: | zero | 1 | 2 | 3 |
| 12. | $\int e^x(x+1) dx$ is equal to: | $e^x + c$ | $xe^x + c$ | $x^2 e^x + c$ | $\frac{xe^x}{2} + c$ |
| 13. | $\int_0^{\pi} \cos x dx$ is equal to: | 2 | -1 | zero | 2 |
| 14. | If $V = x^3$ then differential of V is: | $3x^2$ | $3x^2 dv$ | $x^3 dv$ | $3x^2 dx$ |
| 15. | If $f'(c) = 0$, then f has relative minima at C if $f''(c)$ is: | negative | zero | any value | positive |
| 16. | $\frac{d}{dx}(\sin^{-1} x)$ is equal to: | $\frac{-1}{\sqrt{1-x^2}}$ | $\frac{1}{\sqrt{x^2-1}}$ | $\frac{1}{\sqrt{1-x^2}}$ | $\frac{-1}{\sqrt{x^2-1}}$ |
| 17. | The general solution of $\frac{1}{x} \frac{dy}{dx} - 2y = 0$ is: | $y = ce^x$ | ce^x | ce^{-x} | $ce^{\frac{1}{x^2}}$ |
| 18. | $\frac{d}{dx} \left[\sin \left(\frac{1}{x} \right) \right]$ is equal to: | $x \cos \frac{1}{x}$ | $\frac{-1}{x^2} \cos \frac{1}{x}$ | $\frac{1}{x^2} \cos \frac{1}{x}$ | $\frac{-1}{x} \cos \frac{1}{x}$ |
| 19. | $\lim_{n \rightarrow \infty} \left(1 + \frac{1}{3n} \right)^n$ is equal to: | $e^{\frac{1}{3}}$ | e | e^2 | e^3 |
| 20. | $x = 0$ is solution of inequality: | $2x - 1 < 0$ | $2x + 1 < 0$ | $x < 0$ | $x < -1$ |

Note: - Section I is compulsory. Attempt any three questions from section II.

Section - I

(8 x 2 = 16)

2. Write short answers to any Eight parts.

- i. Given $f(x) = x^3 - ax^2 + bx + 1$. if $f(2) = -3$ and $f(-1) = 0$. Find the value of a and b .
- ii. Find $f^{-1}(x)$ if $f(x) = (-x+9)^3$
- iii. Express the $\lim_{n \rightarrow +\infty} \left(1 + \frac{3}{n}\right)^{2n}$ in terms of "e".
- iv. Evaluate $\lim_{x \rightarrow 0} \frac{1 - \cos 2x}{x^2}$
- v. Differentiate $\left(\sqrt{x} - \frac{1}{\sqrt{x}}\right)^2$ w.r.t "x"
- vi. Find $\frac{dy}{dx}$ if $x = at^2$ and $y = 2at$
- vii. Prove that $\frac{d}{dx}(\cos^{-1} x) = \frac{-1}{\sqrt{1-x^2}}$
- viii. Find $\frac{dy}{dx}$ if $y = x \cos y$
- ix. Find $\frac{dy}{dx}$ if $y = e^{-x}(x^3 + 2x^2 + 1)$
- x. Find $\frac{dy}{dx}$ if $y = e^{-2x} \cdot \sin 2x$
- xi. Find y_2 if $y = 2x^5 - 3x^4 + 4x^3 + x - 2$
- xii. Apply the Maclaurin series expansion to prove that $e^x = 1 + x + \frac{x^2}{2} + \frac{x^3}{3} + \frac{x^4}{4} + \dots$

3. Write short answers to any Eight parts.

(8 x 2 = 16)

- i. Find dy if $y = x^2 - 1$, when x changes from 3 to 3.02
- ii. Evaluate $\int \frac{3x+2}{\sqrt{x}} dx$ ($x > 0$)
- iii. Evaluate $\int \frac{1-x^2}{1+x^2} dx$
- iv. Evaluate $\int \frac{1}{x \ln x} dx$
- v. Evaluate $\int x^4 \ln x dx$
- vi. Evaluate $\int_{-1}^1 (x^{1/3} + 1) dx$
- vii. Find the area above the x -axis and under the curve $y = 5 - x^2$ from $x = -1$ to $x = 2$
- viii. Solve the differential equation $\frac{dy}{dx} = \frac{y}{x^2}$
- ix. Find an equation of line through $A(-6, 5)$ having slope 7.
- x. Find the lines represented by $x^2 - 2xy \sec \alpha + y^2 = 0$
- xi. The points $A(-5, -2)$ and $B(5, -4)$ are ends of diameter of a circle. Find the centre of that circle.
- xii. Check whether the point $(-7, 6)$ lies above or below the line $4x + 3y - 9 = 0$

(Turn Over)

4. Write short answers to any Nine parts.
- Graph the solution set of the linear inequality in xy -plane given by $2x + y \leq 6$.
 - Find the equation of the circle with ends of a diameter at $(-3, 2)$ and $(5, -6)$.
 - Show that the circles $x^2 + y^2 + 2x - 2y - 7 = 0$ and $x^2 + y^2 - 6x + 4y + 9 = 0$ touch externally.
 - Find the length of the chord cut off from the line $2x + 3y = 13$ by the circle $x^2 + y^2 = 26$.
 - Write the equation of a parabola with focus $(-3, 1)$ and directrix $x = 3$.
 - Find the centre and foci of $\frac{x^2}{4} - \frac{y^2}{9} = 1$.
 - Find a unit vector in the direction of the vector $\underline{v} = [-2, 4]$.
 - Find equation of normal to the curve $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ at $(a \cos \theta, b \sin \theta)$.
 - Find α so that $|\alpha \underline{i} + (\alpha + 1)\underline{j} + 2\underline{k}| = 3$.
 - Find the cosine of the angle between $\underline{u} = [2, -3, 1]$ and $\underline{v} = [2, 4, 1]$.
 - Compute the product $\underline{a} \times \underline{b}$, $\underline{a} = -4\underline{i} + \underline{j} - 2\underline{k}$ and $\underline{b} = 2\underline{i} + \underline{j} + \underline{k}$.
 - Find α so that the vectors $\alpha \underline{i} + \underline{j}$, $\underline{i} + \underline{j} + 3\underline{k}$ and $2\underline{i} + \underline{j} - 2\underline{k}$ are coplaner.
 - Find equation of tangent to $x^2 - 2y^2 = 2$ through $(1, -2)$.

Section - II**(Each question carries 10 marks)**

5. (a) Evaluate $\lim_{\theta \rightarrow 0} \frac{\tan \theta - \sin \theta}{\sin^3 \theta}$ 5
- (b) If $x = a \cos^3 \theta$, $y = b \sin^3 \theta$, show that $a \frac{dy}{dx} + b \tan \theta = 0$ 5
6. (a) Evaluate $\int \frac{\sqrt{2}}{\sin x + \cos x} dx$ 5
- (b) Find an equation of the line through the intersection of the lines $x + 2y + 3 = 0$, $3x + 4y + 7 = 0$ and making equal intercepts on the axes. 5
7. (a) Solve the differential equation $\sec x + \tan y \frac{dy}{dx} = 0$ 5
- (b) Minimize $z = 3x + y$; subject to the constraints:
 $3x + 5y \geq 15$; $x + 6y \geq 9$ $x \geq 0$; $y \geq 0$ 5
8. (a) Write equation of circle that passes through the points $A(5, 6)$, $B(-3, 2)$ and $C(3, -4)$. 5
- (b) Prove that: $\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$ 5
9. (a) If $y = e^x \sin x$, then show that $\frac{d^2 y}{dx^2} - 2 \frac{dy}{dx} + 2y = 0$ 5
- (b) Find centre, foci, eccentricity and vertices of $\frac{y^2}{16} - \frac{x^2}{9} = 1$ 5